The Music Lesson

"...the aim and final reason...of all music...should be none else but the Glory of God and the recreation of the mind." (J.S.Bach: 1685-1750)

***Abstract**: Music forms an essential component of the human psyche; appropriately, it is here linked with the Creator's Name, Jesus Christ, and with the opening words of the Judeo-Christian Scriptures.*

1 - Preamble

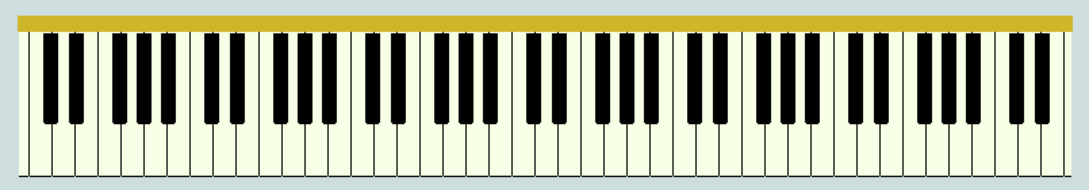
Judged simply as a number, 1.05946309... is hardly likely to stir the emotions. Yet it lies at the heart of that which now - in one form or another - possesses this peculiar quality in abundance. In an age in which ***naturalistic speculation*** in high places is presented as ***irrefutable fact*** it needs to be firmly stated that no account of origins that fails to find a satisfactory explanation for man's interest in music can - or should - be taken seriously. The universality, durability, intensity and variety of this interest are proof enough that music is much more than mere pastime: indeed, life would be unthinkable without it, and its mysterious influence appears to have its roots in the very bedrock of existence.

It is the purpose of this page to draw attention to some of Western music's objective characteristics so that the reality of its close links with our Creator, Jesus Christ, and with the Act of Creation, may become clear.

[Please note. Assistance in this matter is provided in the author's book, ***The Second Edge*** which may be freely downloaded from www.whatabeginning.com/book.pdf; this is referenced as 'SE' in the subsequent text. The numbers represented by Bible's first 8 Hebrew words and their respective geometries may be found at SE, p.30]

2 - The Piano Keyboard

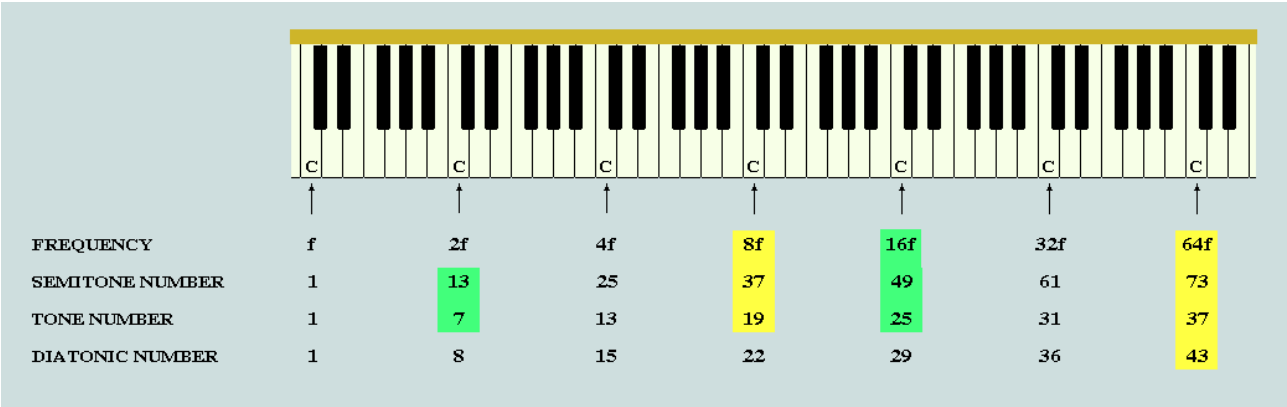
Our formal musical education may well have involved a keyboard of 6 or so octaves, as represented here:



A cursory study of this layout reveals a number of simple facts, viz

- the repeating pattern of 12 keys - 7 white, and 5 black - is termed an ***octave***
- the ***note*** sounded when any key is depressed is said to possess a certain ***pitch*** - this related to the principal ***frequency of vibration*** of the string(s) associated with that particular key
- these frequencies ***double*** at each octave
- the ratio of the frequencies of adjacent notes is uniform, and hence equal to the ***twelfth root of 2*** ($^{12}\sqrt{2}$) - or 1.05946309...
- this indivisible step in pitch is termed a ***semitone***
- each complete octave therefore comprises 12 semitones, or 6 tones
- within each octave, the 7 white notes are named using the alphabetic sequence A - G

These facts are assimilated in the following diagram where bottom ***C*** has been taken as the ***reference note*** - having an assumed principal frequency of ***f*** units:

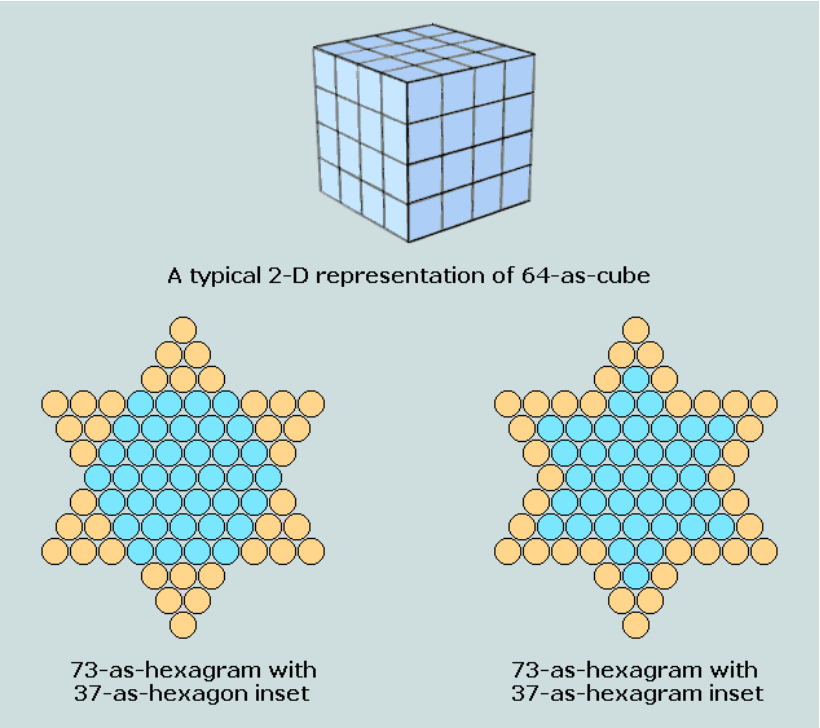


Assuming all notes to have been numbered sequentially from the point of reference (bottom C), a frequency may be assigned to each note that begins a subsequent octave. At the same points, the current counts are shown - for the semitones (12 per octave);for the tones, or semitone-pairs (6 per octave); and for the notes which participate in the diatonic scale of C major (in this case, all the white notes, totalling 7 per octave). Clearly, these relationships hold good throughout the keyboard and are completely independent of the choice of starting position.

There are some familiar numbers represented here which arise from a fair numerical reading of the Bible’s opening words.

The following data arise from the fair numerical reading of the Bible's first 8 words:

- Highlighted in yellow, at the sixth octave, we find the conjunction {64, 73, 37, 43}, and are reminded particularly of the following 'creation geometries':



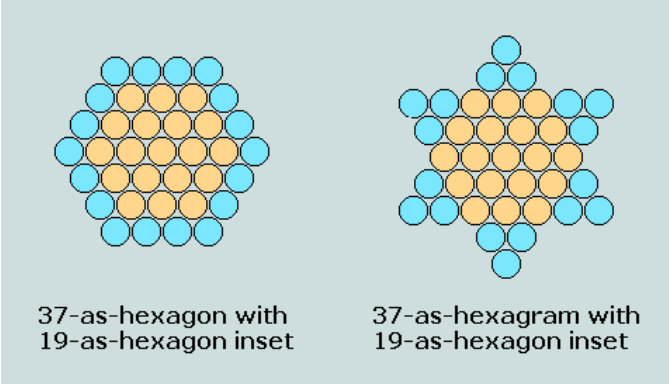
Observe,

(a) in what must be considered a *typical* representation of this cube, one or more faces of precisely 37 of the 64 component unit cubes are visible; and that the product of what is 'visually apparent' by what is 'actual' is 37 x 64, or 2368 - the Creator's number [SE, p.56].

(b) 37 x 73 - another product of related figures - generates 2701, or Genesis 1:1 [SE, pp.19,21].

The only non-geometrical item, 43, is highest prime factor of 86 - value of 'elohim' (i.e. God), the 3rd word of Genesis 1:1 - representing a further reference to the Creator.

- Highlighted in yellow, at the third octave, we find the conjunction {8, 37, 19}, and are reminded of the following:



Observe,

(a) these two primes, 19 and 37, are the factors of 703 - 37th triangular number and sum of words 6 and 7 of Genesis 1:1 [SE, pp.24,30].

(b) the product (8 x 37) = 296 = 7th word of Genesis 1:1 and factor of both the Lord's Name and Title, viz 888 and 1480; it is also distinguished by the fact that it represents the difference between the cubes of 8 (= 512) and 6 (= 216). [SE, pp.30,56].

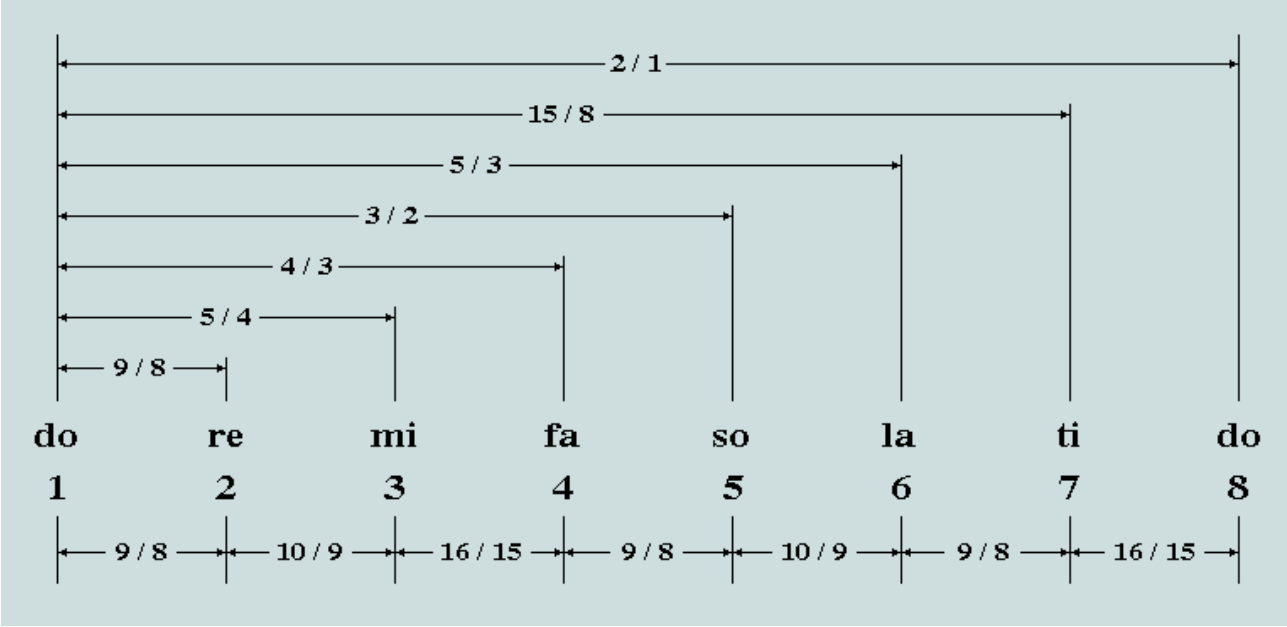
Further details of these conjunctions may be found here.

- Highlighted in green, at the fourth octave, we find the conjunction of three squares, 16, 25 and 49 - the two latter marking the position of the centroid counter of the Genesis 1:1 triangle, 2701; further, 25-as-rhombus and 49-as-rhombus are generators of the hexagon/hexagram pairs 19/37 and 37/73, respectively
- Highlighted in green, at the second octave, we find 7 and 13 - the factors of 91, sole trifigurate companion of 37

Further details of these conjunctions may be found here.

3 - Western Music: a concise background

It was Pythagoras (c.600 BC) who first realised that music, at its deepest level, is mathematical in nature. He saw that a musical scale derives from the mapping of certain proportions onto the indefinite continuum of musical pitch. Thus, as observed above, the eighth of a rising succession of notes in any major or minor scale will be found to have exactly twice the frequency of the first and, sounded together, they are said to be *in unison*. The interval of pitch represented is called 'an eighth' or 'octave'. Intermediate intervals are similarly described, e.g. that between the first and fifth notes is called 'a fifth'. In an ideal ***natural scale*** the frequency ratios are *vulgar fractions*, thus:

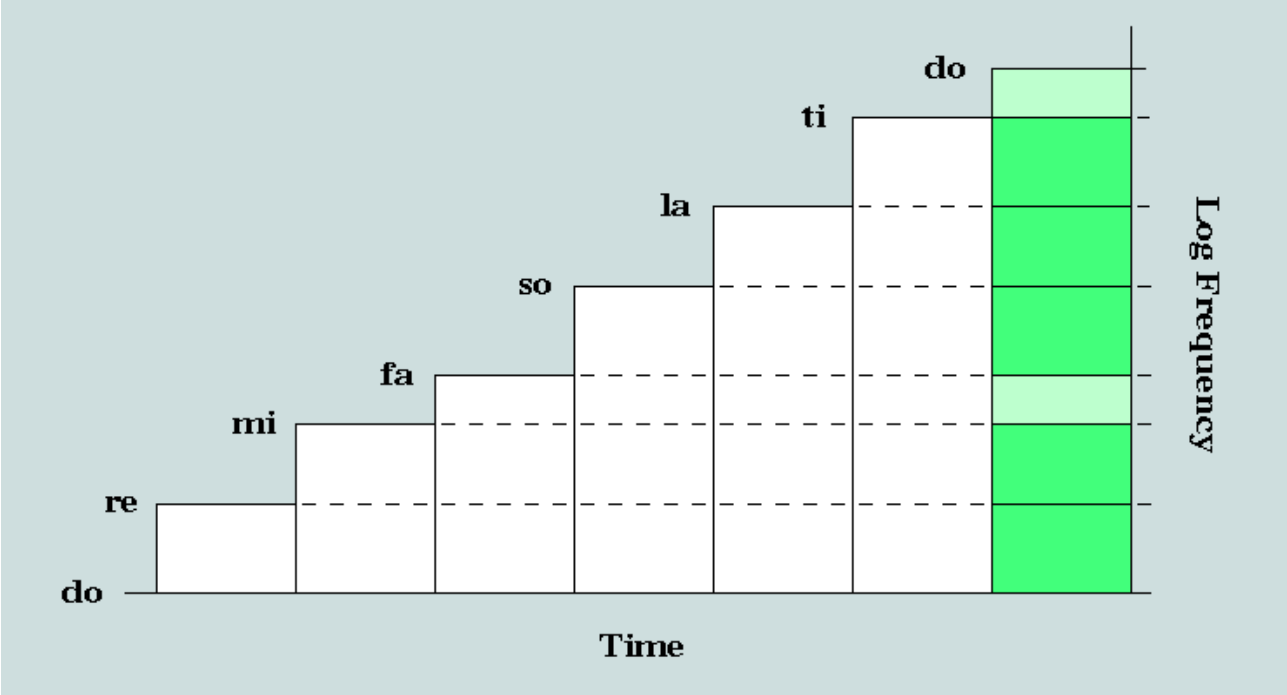


The 'Just Intonation' or 'Natural' Scale

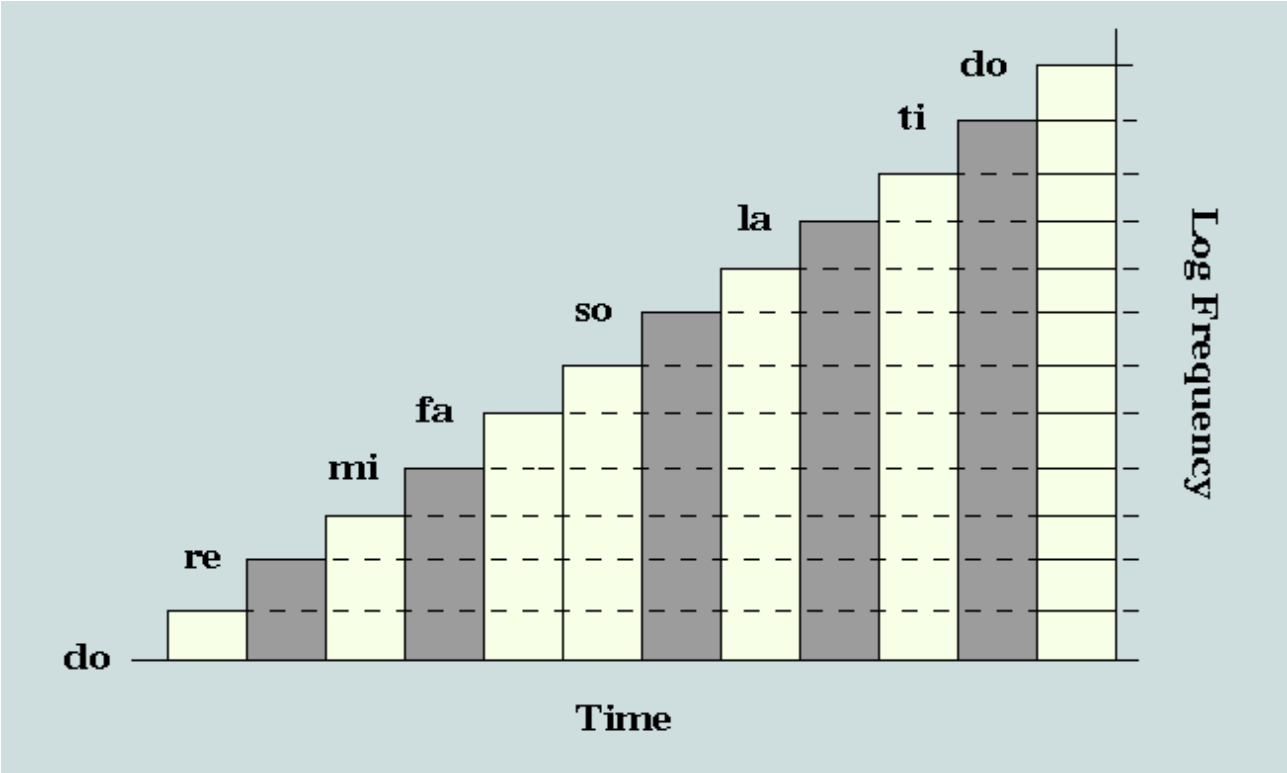
The notes of the scale are named and numbered in the usual way and, by setting C = do, the scale may be played entirely on the white notes of a keyboard. Irrespective of absolute pitch, or key, a succession of notes having these frequency ratios will be instantly recognised by the normal human ear as a ***major scale***. The relationships, of course, extend above and below the selected octave - the same ratios relating to the corresponding octave notes. Any attempt to to play a major scale from some other point in this sequence must fail because the succession of intervals is then different. Indeed, to allow this scale to be played in any other key would require the availability of very many additional notes. Clearly, the limitations imposed by the natural scale do not permit harmony and modulation in the fullest sense*****. Hence, early music - so strange-sounding to the modern ear was largely *monodic* i.e. consisted of a single melody line; it recognised a system of ***modes*** which provided the basic material of Western music well into the Middle Ages.

*** However, it is interesting to observe that a group of choristers singing without instrumental accompaniment will tend to harmonise automatically using these natural intervals; similarly, instruments that can bend pitches enough to fine-tune them during a performance - and this includes most orchestral instruments - also tend to play "pure" intervals. But for many instruments that cannot be fine-tuned quickly - such as piano, organ and harp - *tuning* becomes a big issue.**

It is helpful to present the foregoing data in the form of a 'Log Frequency/Time' graph, as follows:



Observe that the steps 'mi-fa' and 'ti-do' are approximately half those of the remaining 5 intervals. A logical development (which reputedly occurred in the late Sixteenth Century) was to halve each of these wider intervals so as to divide the octave into a ***chromatic scale*** of 12 ***semitones***, thus:



And so our familiar keyboard was born!

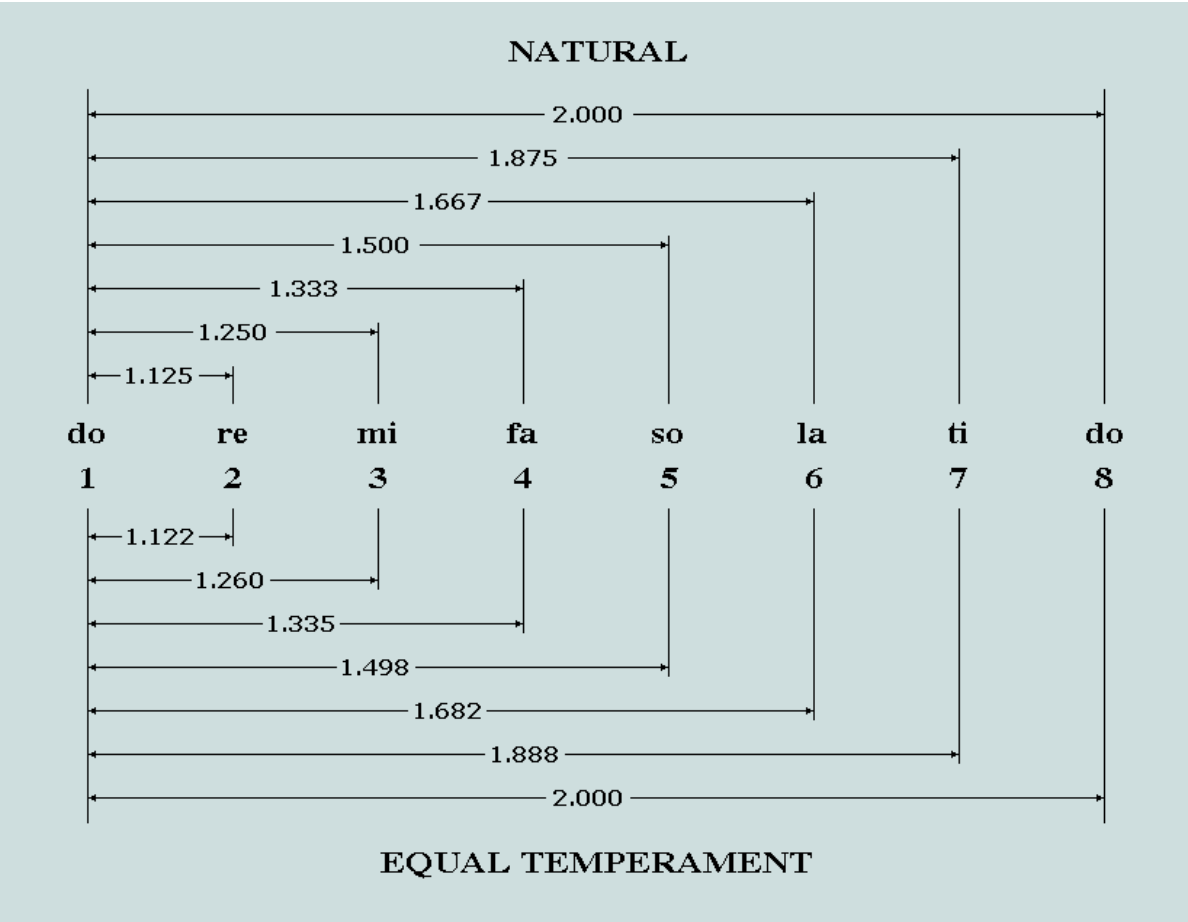
A glance at the rightmost column, however, reveals that the division was not uniform; in other words, the frequency ratio between successive semitones varied. Thus, the moves to free music from its natural 'straightjacket' had also to include adjustments to the frequency ratios in this 'chromatic' scale so that one could play equally well in all keys.

Many so-called ***meantone*** tunings were devised to achieve this - the aim always being to minimise the inevitable clashes, whilst yet retaining the individual 'character' of each key. For example, in respect of Bach's *The Well-Tempered Clavier* - two sets of 24 Preludes and Fugues in all major and minor keys - it is highly probable that the form of tuning known as 'Werckmeister III' provided a satisfactory basis.

[Readers interested in the finer points of instrument tuning may find Nigel Taylor's tuning page at www.ringing.info/nigel-taylor/index.html]

Equal temperament - the ultimate, bland, ***equal spacing*** of the 12 pitches of the octave - is essentially a Twentieth-Century phenomenon and, in Western music, is now considered the ***standard*** - even for voice and those instruments capable of being played in just intonation. Observe that in equal temperament, the only *pure* intervals are the octaves. This form of tuning is well suited to music that changes key often, is very chromatic, or is harmonically complex. Nevertheless, because it lacks pure intervals, and because the individual character of each key is lost, many musicians find it unsatisfying.

In the following diagram, a comparison is made between natural and equal temperament tunings - the numbers representing the frequency ratios between major scale notes.



Observe that whilst most of the intervals match reasonably well, there are problems with the 6th and 7th degrees of the scale where, as may be seen, errors approaching +1% are incurred:

DEGREE (MAJOR SCALE)	NATURAL	EQUAL TEMPERAMENT	ERROR
1 = do	1.000	1.000	+0.00%
2 = re	1.125	1.122	-0.26%
3 = mi	1.250	1.260	+0.80%
4 = fa	1.333	1.335	+0.15%
5 = so	1.500	1.498	-0.13%
6 = la	1.667	1.682	+0.90%
7 = ti	1.875	1.888	+0.69%
8 = do	2.000	2.000	+0.00%

Expressed mathematically, the audio frequencies associated with any equally-tempered chromatic scale form a ***geometric progression*** - first term (**f**): some arbitrary reference frequency; and common ratio (**r**): the twelfth root of 2 [$^{12}\sqrt{2}$].

Observe that a good approximation of $^{12}\sqrt{2}$ may be obtained from Genesis 1:1 as the ratio 105/99 = (4^{th} word – 7^{th} word) / (5^{th} word – 7^{th} word), i.e.

(401 – 296) / (395 – 296) = 1.0606… (error: +0.1%)

Further observe that $\sqrt{2}$ (= $\{^{12}\sqrt{2}\}^6$) is even better represented as the ratio

297 / 210 = 3(395 – 296) / (296 – 86) … (error: +0.005%)

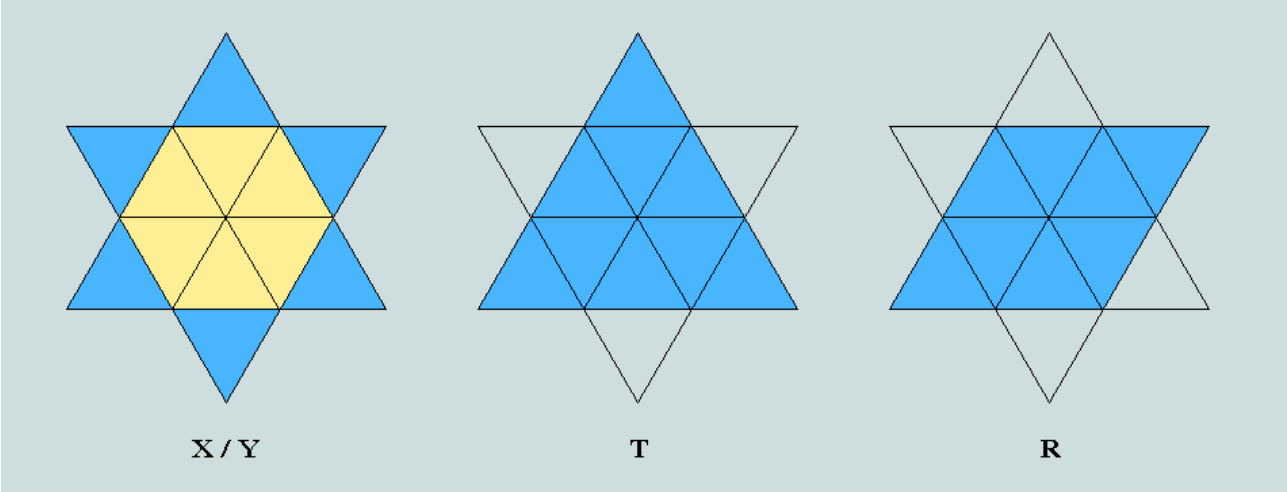
This also relates Genesis 1:1 to the ISO standard A4 sheet of paper:

(see www.whatabeginning.com/A4Enigma.pdf)

4 - The Geometrical Analogues

3.1 - Just Intonation - I

The principal frequency ratios of the natural scale may be expressed very simply and elegantly in geometric terms, as follows:



A geometrical expression of the natural scale ratios

The basis of the analogy is the symmetrical hexagram (**Y**) constructed from 12 identical equilateral triangular tiles. Central to this figure is the symmetrical hexagon (**X**) - comprising 6 of these elements. The large triangle (**T**), and the rhombus (**R**), which are each capable of generating **X** / **Y** (by self-intersection/self-union, following one rotational step of 180 degrees, or two of

60 degrees, respectively) incorporate 9, and 8 of these elemental tiles, respectively.

Linking these observations with the just intonation ratios provided above, we may write:

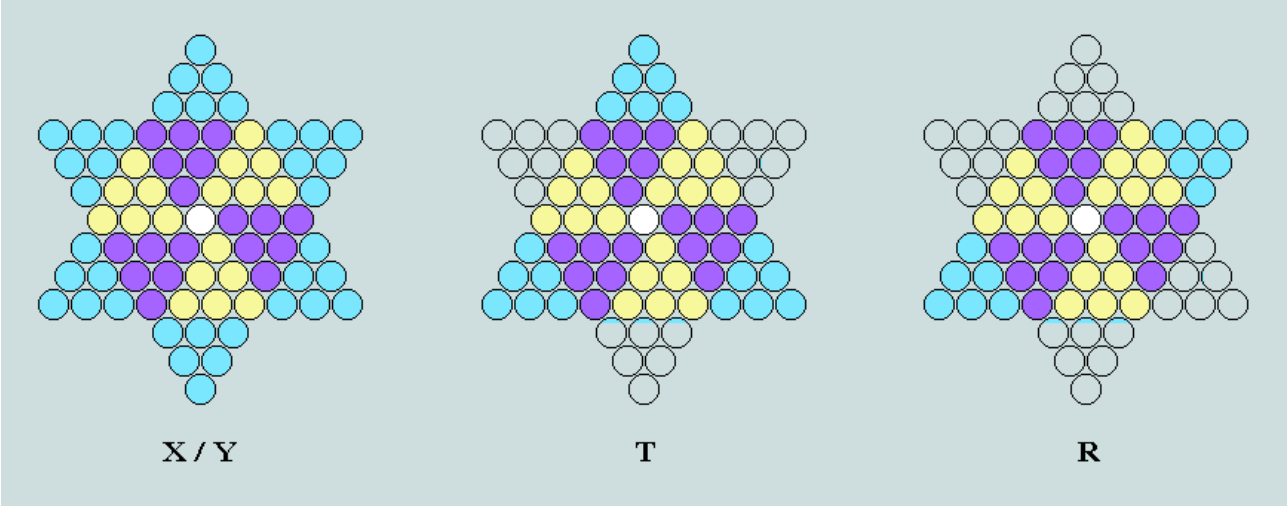
- Y:X = 2:1 = the eighth, or octave

- T:X = Y:R = 3:2 = 1.500 :1 = the fifth
- R:X = Y:T = 4:3 = 1.333 :1 = the fourth
- T:R = 9:8 = 1.125 :1 = the second

Clearly, we witness here a marked correspondence with the chief intervals of the natural scale.

3.2 - Just Intonation - II

But such geometries may also appear in an alternative 'digitised' form - the elemental triangles now replaced by standard representations of *triangular numbers*. Consider, for example,



In this illustration, observe that the 12 elemental triangles of the hexagram (**Y**) - each of 6 units - are set around a single 'centroid' counter. Clearly, the totals of units for this and the associated figures are as follows:

Y = 12x6 + 1 = 73; X = 6x6 + 1 = 37; T = 9x6 + 1 = 55; R = 8x6 + 1 = 49

and their ratios:

- Y:X = 73:37 = 1.973 :1 ≈ 2:1 (error = - 1.35%)

- T:X = 55:37 = 1.486 :1 ≈ 3:2 (error = - 0.93%)
- R:X = 49:37 = 1.324 :1 ≈ 4:3 (error = - 0.70%)
- T:R = 55:49 = 1.122 :1 ≈ 9:8 (error = - 0.27%)

Clearly, these errors are already of the same order of size as those between the scales of equal temperament and just intonation; with the introduction of larger elemental triangles, they may be reduced - without limit.

3.3 - Equal Temperament

The final analogue concerns the scale of equal temperament (ET) which, as already observed, has become the universal standard tuning procedure. However, before turning to that let us clarify any doubts there may be concerning the two alternative ways of generating hexagram/hexagon (X/Y) pairs – i.e. symmetrical figures which have already featured large in this analysis. The matter is discussed here.

A principal feature of ET is that the octave is divided into 12 equal, indivisible, steps of pitch - the semitones - these, the subjective interpretation of a geometric progression of frequencies, common ratio, $\sqrt[12]{2}$ (=1.05946309...). This is closely approximated by the rational number 196/185 (= 1.05945945...) which differs from the true semitone ratio by a mere 0.00034% (and is therefore negligible when compared with the inherent errors of ET discussed earlier).

Now 196/185 is a fraction of some considerable interest, for observe:

Its numerator, 196 = 2^2 x 7^2 = 14^2 = 4 x 49 = 7 x 28 = 7 x 7th triangular number

= (order of triangle generating 37-as-hexagram) x (that triangle)

= (No. of Words in Genesis 1:1) x (No. of Letters in the same verse)

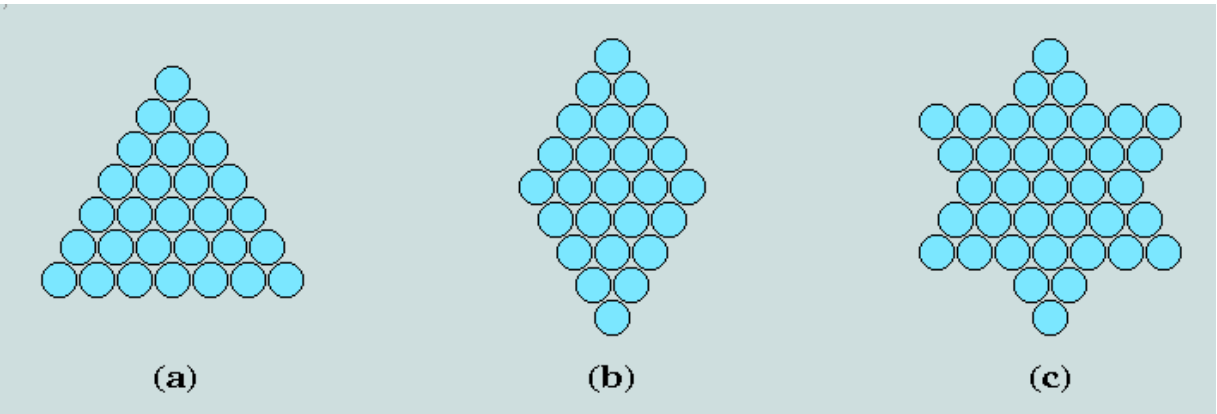
Its denominator, 185 = 5 x 37

= (order of rhombus generating 37-as-hexagram) x (that hexagram)

= [(sum of words 1 to 5) - (sum of words 6 to 7)] / 7

= [1998 - 703] / 7 = 1295 / 7 = 185

For convenience, the geometries referred to above are reproduced here:



At (a) and (b) we have the figures capable of generating the particular hexagram (c)

It is therefore abundantly clear that with the introduction of the scale of equal temperament in the Twentieth Century and its adoption as the tuning standard came these powerful links with the elements of numerical geometry, and with the opening words of the Judeo-Christian Scriptures.

Here is a valuable addition to the properties of the **Creation Cryptogram** which – by God's grace – inhabits and confirms these informative Hebrew words.

5 - Concluding Remarks

The more one contemplates the phenomenon of music, the more remarkable it appears. It not only enables man to express his many moods and to pour forth his innermost thoughts and emotions in a natural way but, as we have seen in this brief anaysis, it enables him through reasoned argument to know his true roots. Is man God's ***Special Creation***, as the Bible maintains, or is he rather a ***cosmological/biological accident***? The answer now seems clear enough! Music reveals this much-disputed truth. In itself, of course, music is *neutral;* it has the ability to uplift or degrade the human spirit. Yet this unique form of expression had its beginnings in a Creation that God pronounced ***very good***. As we listen to a 'dawn chorus', to the "Gloria" from Bach's B minor Mass, or to his "Erbarme dich" from the St.Matthew Passion, can we any longer doubt that the ***true*** purpose of music is to enable His Creation ***to Praise, to Thank, and to Glorify Him***!?

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